Eigen-Direction Alignment Aided Physical Layer Network Coding for MIMO Two-Way Relay Channels

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Related Work

- Tao Yang, Xiaojun Yuan, Li Ping, Iain B. Collings, and Jinghong Yuan, "Eigen-direction alignment aided physical layer network coding for MIMO two-way relay channels", submitted to ISIT 2011.

- Dr. Tao Yang and Dr. Iain B. Collings are with the Wireless & Networking Technology Lab, CSIRO ICT Center, Sydney, Australia.
- Prof. Ping Li is with the Department of Electronic Engineering, City University of Hong Kong, HK SAR.
- Prof. Jinghong Yuan is with the Department of Electrical Engineering, University of New South Wales (UNSW), Sydney, Australia.

Outline

- Channel Model
- Existing Methods
- Eigen-Direction Alignment (EDA) Precoding
- Numerical Results
- Conclusions and Future Work

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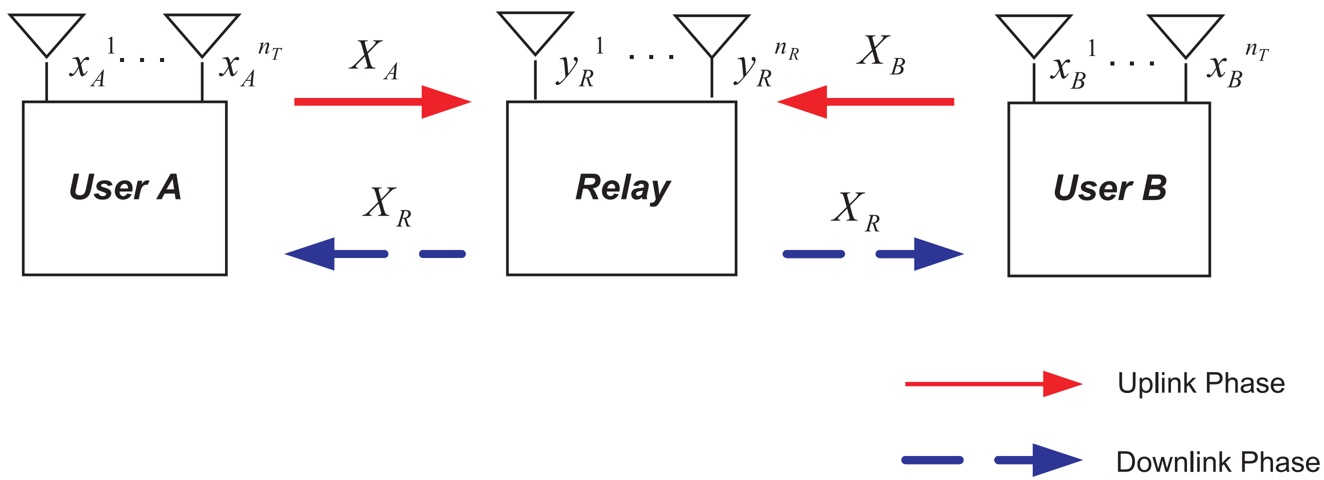
MIMO Two-Way Relay Channel (TWRC)



- Three-node two-phase transmission
- Users A and B equipped with n_T antennas and Relay with n_R antennas
- Uplink phase: $\mathbf{Y}_R = \mathbf{H}_A\mathbf{X}_A + \mathbf{H}_B\mathbf{X}_B + \mathbf{Z}_R$
- Relay operation: $\mathbf{X}_R = f_R(\mathbf{Y}_R)$
- Downlink phase: $\mathbf{Y}_A = \mathbf{H}_A^T\mathbf{X}_R + \mathbf{Z}_A$ and $\mathbf{Y}_B = \mathbf{H}_B^T\mathbf{X}_R + \mathbf{Z}_B$

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Capacity Upper Bound

- From the cut-set bound, the achievable rate pair of the MIMO TWRC channel is upper bounded by

$$R_A^{UB} \le \frac{1}{2}\min\left\{\log\det\left(\mathbf{I}+\mathbf{H}_A\mathbf{Q}_A\mathbf{H}_A^H\right), \log\det\left(\mathbf{I}+\mathbf{H}_B^T\mathbf{Q}_R\mathbf{H}_B^*\right)\right\}$$

$$R_B^{UB} \le \frac{1}{2}\min\left\{\log\det\left(\mathbf{I}+\mathbf{H}_B\mathbf{Q}_B\mathbf{H}_B^H\right), \log\det\left(\mathbf{I}+\mathbf{H}_A^T\mathbf{Q}_R\mathbf{H}_A^*\right)\right\}$$

- This bound can be achieved as if the two-way relay channel can be decomposed into two independent one-way relay channels.

Existing Methods

- Analogue Network Coding (ANC) Scheme
  - Amplify and Forward (AF) at the Relay
    - $\mathbf{X}_R = f_R(\mathbf{Y}_R) = \mathbf{A}\mathbf{Y}_R$
  - At the user end, the known signal component is subtracted from the received signal, and then decoding is performed.

- Disadvantage:
  - Noise amplification
  - Unnecessary power consumption at the relay

S. Xu and Y. Hua, "Source-relay optimization for a two-way MIMO relay system", *Proc. IEEE ICCASP 2010*, pp. 3038-3041.

Existing Methods (Continued)

- Decode-and-Forward and MAC Layer Network Coding (DF-MNC)
  - Full decoding at the relay
  - Network coded signal T transmitted by the relay
  - At the user end, the receiver decodes T with the help of knowing its own transmitted signal.

- Disadvantage
  - Multiplexing loss: Full decoding at the relay is not necessary!

Recent Progress in SISO TWRC

- Physical Layer Network Coding (PLNC) for SISO TWRC
  - Use nested lattice codes
  - Relay only decodes just-enough information T and transmit its coded version in the downlink
  - At each user end, the receiver decodes T with the help of knowing its own signal.

- This scheme can achieve the capacity upper bound within ½ bit

W. Nam, S. Chung, Y. H. Lee, “Capacity of the Gaussian two-way relay channel to within 1/2 bit”, submitted to *IEEE Trans. Inform. Theory*, 2009.

Challenges in MIMO TWRC

- Nested lattice codes requires that the received signal locates on a structured lattice.
- In MIMO TWRC, the signals received by the relay is multi-dimensional in space, and in general, without any regular structure.
- Therefore, nested lattice codes is not directly applicable to MIMO TWRC.
- What can we do next?

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Zero-Forcing (ZF) Precoding

- Assumption: The number of antennas at the user end (n_T) is no less than that at the relay (n_R)

- Uplink channel model: $\mathbf{Y}_R = \mathbf{H}_A\mathbf{X}_A + \mathbf{H}_B\mathbf{X}_B + \mathbf{Z}_R$
- Precoder at User A: $\mathbf{X}_A = \mathbf{H}_A^{-1}\boldsymbol{\Psi}_A\mathbf{C}_A$
- Precoder at User B: $\mathbf{X}_B = \mathbf{H}_B^{-1}\boldsymbol{\Psi}_B\mathbf{C}_B$

- $\boldsymbol{\Psi}_A$ and $\boldsymbol{\Psi}_B$ are diagonal matrices for power allocation
- $\mathbf{C}_A$ and $\mathbf{C}_B$ contain iid coded streams
- Equivalent parallel channels for the uplink:

$$\mathbf{Y}_R = \boldsymbol{\Psi}_A\mathbf{C}_A + \boldsymbol{\Psi}_B\mathbf{C}_B + \mathbf{Z}_R$$

- Performance loss is significant when $\mathbf{H}_A$ and $\mathbf{H}_B$ are ill-conditioned!

Eigen-Direction Alignment (EDA) Precoding

- Uplink channel model: $\mathbf{Y}_\mathrm{R} = \mathbf{H}_\mathrm{A}\mathbf{X}_\mathrm{A} + \mathbf{H}_\mathrm{B}\mathbf{X}_\mathrm{B} + \mathbf{Z}_\mathrm{R}$
- Rotated by a unitary matrix $\mathbf{K}$:

$$\mathbf{K}\mathbf{Y}_\mathrm{R} = \mathbf{K}\mathbf{H}_\mathrm{A}\mathbf{X}_\mathrm{A} + \mathbf{K}\mathbf{H}_\mathrm{B}\mathbf{X}_\mathrm{B} + \mathbf{K}\mathbf{Z}_\mathrm{R}$$

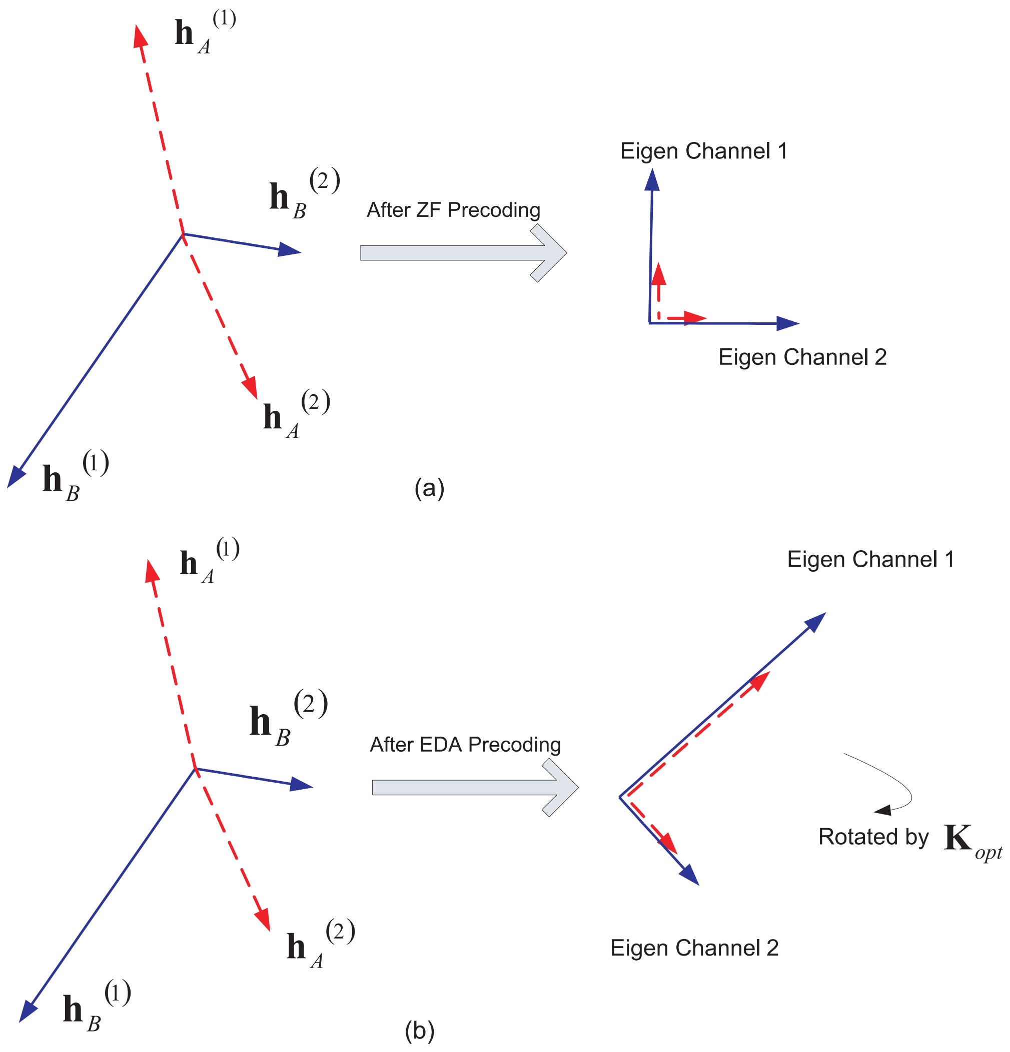
- Precoder at User A: $\mathbf{X}_\mathrm{A} = (\mathbf{K}\mathbf{H}_\mathrm{A})^{-1}\Psi_\mathrm{A}\mathbf{C}_\mathrm{A}$
- Precoder at User B: $\mathbf{X}_\mathrm{B} = (\mathbf{K}\mathbf{H}_\mathrm{B})^{-1}\Psi_\mathrm{B}\mathbf{C}_\mathrm{B}$
- Ψ_A and Ψ_B are diagonal matrices for power allocation
- $\mathbf{C}_\mathrm{A}$ and $\mathbf{C}_\mathrm{B}$ contain iid coded streams

- Equivalent parallel uplink channels:

$$\mathbf{K}\mathbf{Y}_\mathrm{R} = \Psi_\mathrm{A}\mathbf{C}_\mathrm{A} + \Psi_\mathrm{B}\mathbf{C}_\mathrm{B} + \mathbf{K}\mathbf{Z}_\mathrm{R}$$

- With a proper choice of $\mathbf{K}$, we can avoid taking the inverse of a ill-conditioned matrix.

Zero-Forcing (ZR) Vs Eigen-Direction Alignment (EDA)



EDA-Aided Physical-Layer Network Coding (PLNC) for MIMO TWRC

- Uplink phase
  - With EDA, the equivalent parallel channels for the uplink :

$$\mathbf{Y}_{\mathrm{R}} = \Psi_{\mathrm{A}}\mathbf{C}_{\mathrm{A}} + \Psi_{\mathrm{B}}\mathbf{C}_{\mathrm{B}} + \mathbf{Z}_{\mathrm{R}}$$

  - Physical-layer network coding (PLNC) is applied to each parallel SISO channel.
  - Relay decodes the super index T (consisting of the messages from all the uplink parallel channels).

- Downlink phase
  - T is re-encoded and then forwarded to the users.
  - Each user decodes T with the help of knowing its own transmitted signal.

Achievable Rate Pair of EDA-PLNC

- Theorem 1: The following rate pair is achievable by the proposed EDA-PLNC scheme.

$$R_A \leq \frac{1}{2}\min\left\{R_A^{UL}, R_A^{DL}\right\}$$

$$R_B \leq \frac{1}{2}\min\left\{R_B^{UL}, R_B^{DL}\right\}$$

$$R_A^{UL} = \sum_{i=1}^{n_R}\left[\log\left(\frac{\psi_A(i,i)^2}{\psi_A(i,i)^2 + \psi_B(i,i)^2} + \psi_A(i,i)^2\right)\right]^+$$

$$R_B^{UL} = \sum_{i=1}^{n_R}\left[\log\left(\frac{\psi_B(i,i)^2}{\psi_B(i,i)^2 + \psi_A(i,i)^2} + \psi_B(i,i)^2\right)\right]^+$$

$$R_A^{DL} = \log\det\left(\mathbf{I} + \mathbf{H}_B^T \mathbf{Q}_R \mathbf{H}_B^*\right)$$

$$R_B^{DL} = \log\det\left(\mathbf{I} + \mathbf{H}_A^T \mathbf{Q}_R \mathbf{H}_A^*\right)$$

Optimization Problem

- Problem Formulation:

  maximize $\alpha R_A + (1-\alpha) R_B$

  subject to certain power constraints

- Parameters to be optimized:

  rotation matrix $\mathbf{K}$, power allocation matrices Ψ_{A} and Ψ_{B}

- This problem is non-convex, and thus is difficult to solve.

- The optimal solution can be found by imposing $\Psi_{\mathrm{A}} = \gamma \Psi_{\mathrm{B}}$

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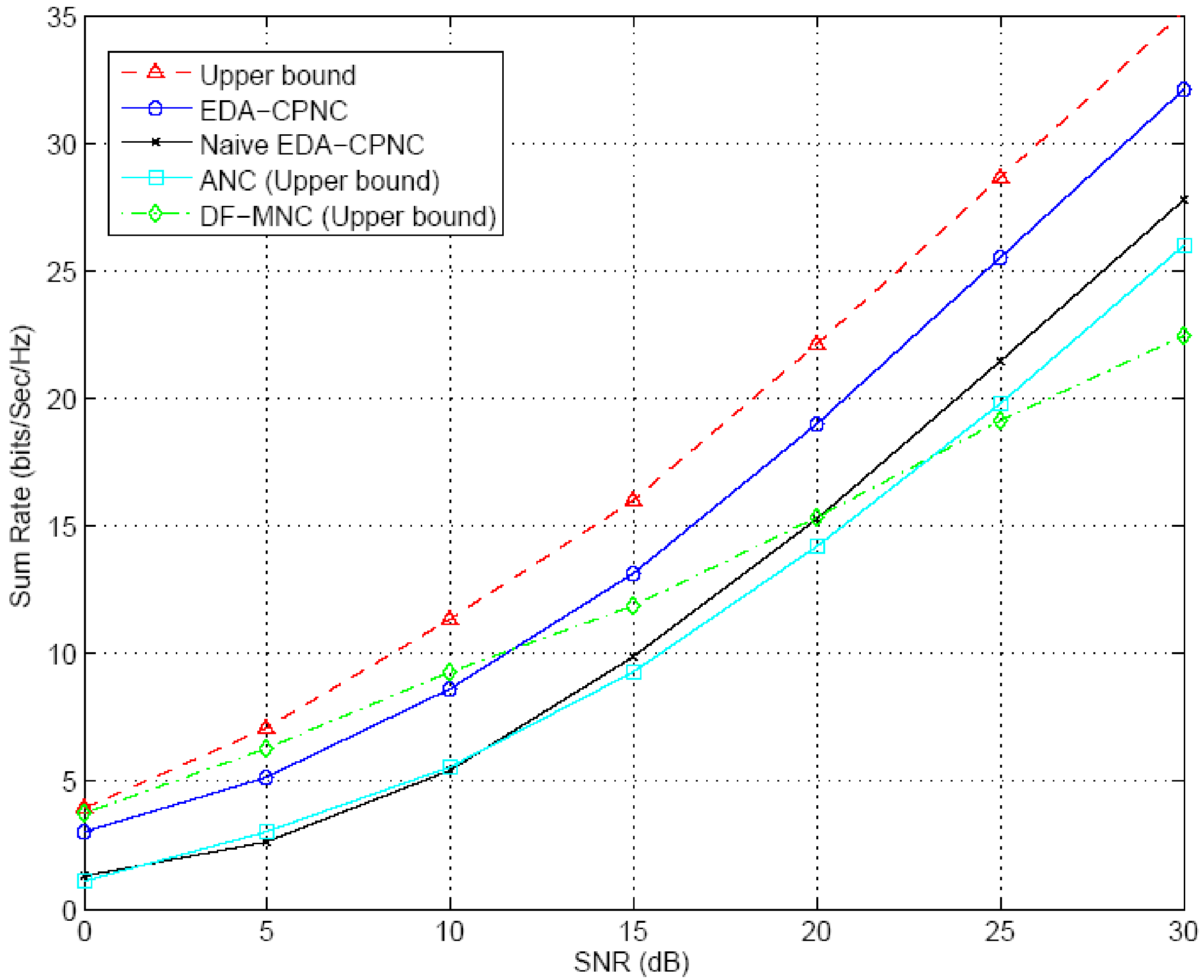
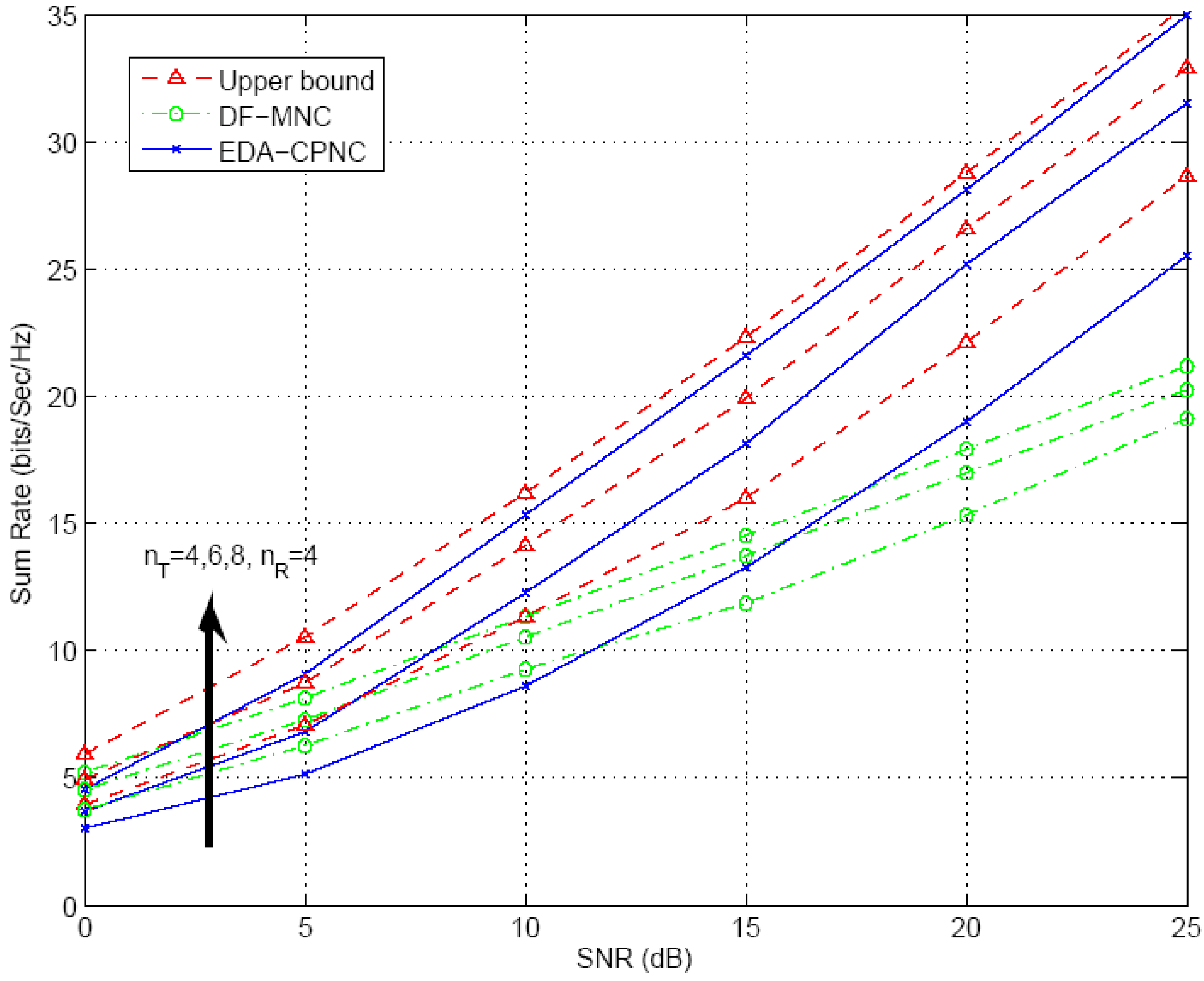
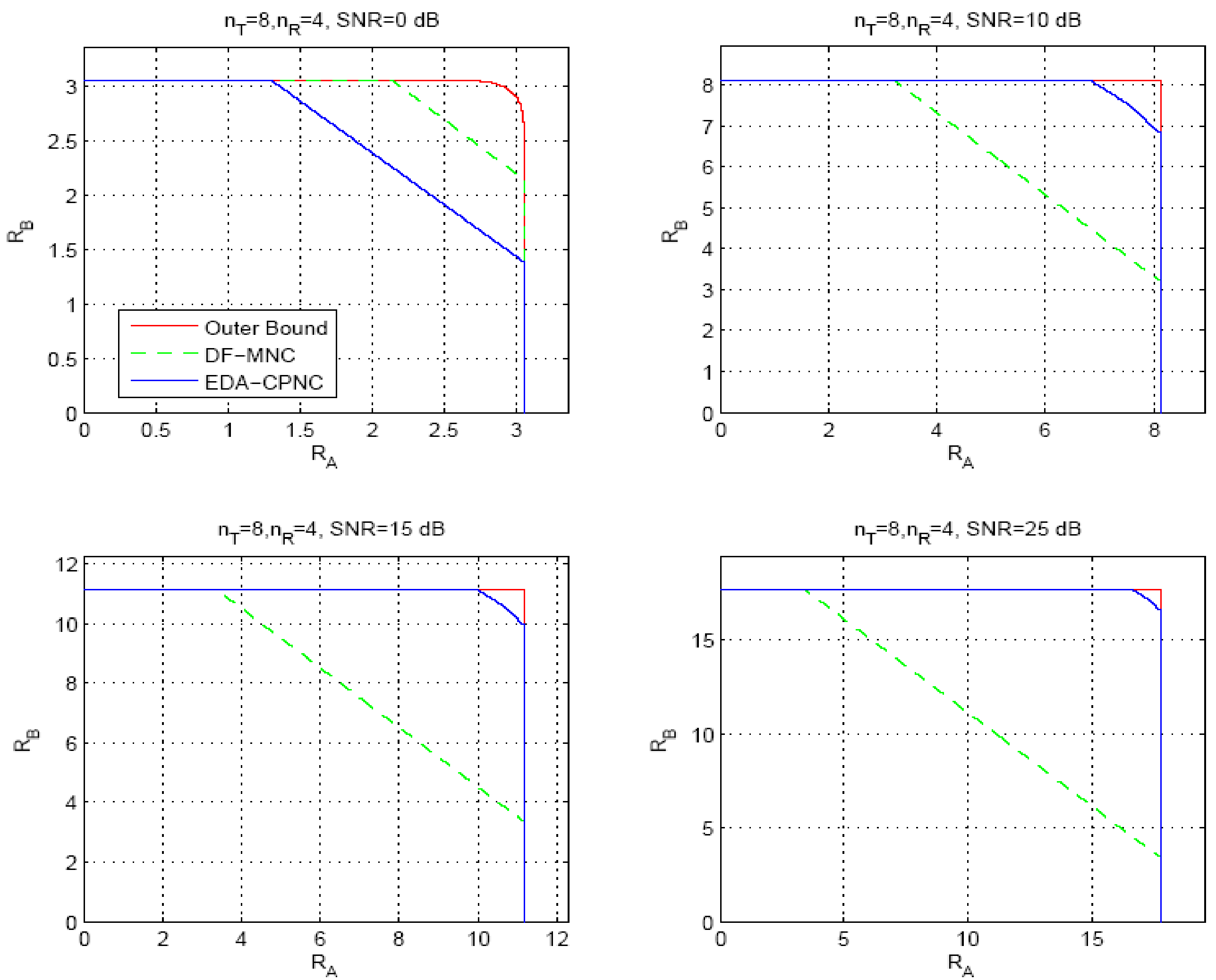
Numerical Results: Sum-Rate (4X4)



Numerical Results: Sum-Rate



Numerical Results: Capacity Region (8X4)



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Conclusions

- We propose an eigen-direction alignment (EDA) technique to efficiently create a set of parallel channels for MIMO TWRC.
- We show that the EDA-aided PNLC scheme can approach the capacity upper bound of MIMO TWRC.

Future Work

- Find the optimal solution to the general case without imposing

$$\Psi_{\mathrm{A}} = \gamma\Psi_{\mathrm{B}}$$

- Generalize the results to the case of

$$n_{\mathrm{T}} < n_{\mathrm{R}}$$

Thank you !